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TEVLAT: A NEW PROGRAM FOR COMPUTING LATTICE FUNCTIONS FOR THE ENERGY SAVER/DOUBLER

A. D. Russell

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The superconducting magnets to be used in the Energy Saver/Doubler present the accelerator designer with many unique problems. Prominent among these problems is the need to understand the effects of the unusually rich multipole structure of the magnets. To use existing general-purpose programs, such as SYNCH or PATRICIA, in this situation requires substantial special coding. In addition the many special features of these programs which are not needed impose an unnecessary overhead. It was therefore decided to develop a new program that is designed specifically to meet the needs of the ES/D project. Advantages to be expected include computational efficiency, ease of use, and the development of a set of sub-routines that may also be useful elsewhere in the project.

The Program

TEVLAT is essentially a conventional ray-tracing program. The design dipole and quadrupole fields are described in a form equivalent to the conventional matrix formulation.[1] Corrections due to the other multipole components in the fields of the dipole and quadrupole magnets are included as impulses applied at the center of the element. The correction magnets are described as impulses located as specified in the Design Report.[2]

The nonlinearities and couplings introduced by the skew and higher normal multipole components of the fields of the dipole and quadrupole magnets are always essential to the description of the ES/D lattice. For this reason, one cannot define a transfer matrix which is independent of a particle's initial phase space coordinates to carry it from point-to-point in the ring. Each particle must be traced, element-by-element, through the lattice on each turn.

The first step in computing the lattice functions is to determine a closed orbit at a given momentum. (Momenta are expressed as a fractional deviation, $\delta p/p$, from the nominal design momentum.) The closed orbit is determined by propagating a particle through the lattice, starting at the middle of the CO long straight section, and iteratively correcting the starting phase-space coordinates until the particle returns to within an acceptably small distance (in phase space) of its starting point. The displacement of the closed orbit from the design orbit gives immediately the momentum dispersion function.

The betatron functions are computed by examining the small amplitude betatron oscillations of the particle about the closed orbit. The radial and vertical betatron oscillations are coupled by normal multipole components of order higher than quadrupole and by any skew multipole components in the fields of the magnets. (When considering only normal multipoles, the coupling is due entirely to motion of the perturbed particle out of the plane of symmetry. The effect becomes more significant as $|\delta p/p|$ increases.) The analysis is therefore formulated in terms of the

4×4 small oscillations transfer matrix described by Courant and Snyder.[3] When the modes of oscillation are not coupled, this formulation is strictly equivalent to the more common 2×2 matrix analysis.

The small amplitude betatron oscillations are stable if the eigenvalues, λ_i, of the transfer matrix are complex conjugate pairs lying on the unit circle. The fractional part of the tunes of the two modes of oscillation are then given by $\lambda_i=\exp(i\mu_i)$.

The definitions of the betatron function α, β, and γ in terms of the elements of the transfer matrix are well known when the motions are uncoupled. To properly define these functions when the oscillations are coupled is rather more complicated. One must determine the normal modes of the system, usually by formulating the problem in terms of a Hamiltonian and then performing a series of canonical transformations. For the work reported here I have taken a less sophisticated empirical approach. The tunes are properly defined by the eigenvalues of the transfer matrix and then the same combinations of matrix elements as in the uncoupled case are used to define α, β, and γ. When the coupling is small, as is certainly the case when only normal multipoles are included, this approximation may be expected to be reasonably serviceable.

TEVLAT has been written for use on the Fermilab CYBER 175 system in either interactive or batch mode. Field length required to run the program is 72000 (octal) words. The minimum size of the program is limited by the need to have available in core the values of 12 multipole coefficients for each of 1200 magnets, using 34100 (octal) words. To minimize resource useage, the program is loaded with a segmented map.

Output from TEVLAT consists of summaries of the closed-orbit parameters, the betatron functions, and the small oscillation transfer matrices. Optionally, all the functions can be plotted, either directly on a graphics CRT terminal or remotely on the Calcomp plotter.

Tests of the program show that it requires approximately 0.11 CPU seconds to compute one particle-turn. In a test run, TEVLAT took 40.976 CPU seconds to complete a full set of calculations at 21 momenta. The calculations required an average of 17.1 turns at each momentum. The total time required for all calculations other than propagating particles through the lattice was determined to be 1.070 CPU seconds. No plots were generated by this run.

The lattice functions are now calculated at only one point in the lattice, the middle of the CO long straight section. Modifications to calculate the functions around the lattice will require eight more turns per momentum. Less than 3 CPU seconds should be required to calculate a complete set of lattice functions at one momentum.

Results

We present here results from a suite of examples that were prepared to illustrate the use of TEVLAT and to establish a connection with earlier work. These results are presented as plots of the tunes, ν_x and ν_y, of the radial (x-direction) and vertical (y-direction) betatron oscillations versus fractional deviation, $\delta p/p$, from the design momentum. Only normal multipoles are included so the closed orbits lie in the y=0 plane of symmetry.

Previous work on these problems is summarized in the Design Report.[2] Comparisons with the SYNCH runs prepared by D. Johnson show complete agreement except for small differences attributable to the neglect of fringe fields in the present work.

Figure 1 shows the results for a purely linear machine. The chromaticity is -22.5. The calculations have been extended to larger values of $\delta p/p$ and are limited only by the inability to find a closed orbit as the tunes approach integers. No physical aperture limits are included in the program.

Figures 2 through 19 were generated by adding, one by one, quadrupole through 38-pole components of strength $b_n = 1\times 10^{-4}$ conventional units to the dipoles. The qualitative features of the results are as expected. The quadrupole term splits the tunes and the sextupole term changes the chromaticities. Progressively, the effects of the higher multipoles are restricted more and more to larger values of $\delta p/p$ where the closed orbits are farther from the design orbit.

The quadrupole example, Fig. 2, shows tunes shifts $\Delta\nu_x = 0.112$ and $\Delta\nu_y = -0.115$ with respect to the linear machine. Using the expression[3]

$$\Delta\nu = \frac{1}{4\pi} \int_0^C \beta(s)\, k(s)\, ds$$

and the SYNCH results in the Design Report[2] we estimate $\Delta\nu = 0.118$, and the shifts are in proper directions.

Figure 20 shows the effect of including the design multipole components (the "Snowdon fields") in the dipoles. No chromaticity corrections are included in this example. The calculations show that the betatron oscillations are stable throughout the interval shown. The limits on the calculation are imposed by the inability to find closed orbits as the tunes approach integer values at large $\delta p/p$. The locations and qualitative features of the limits of the momentum aperture are consistent with results obtained by M. Harrison[4] in his Monte Carlo studies of extraction.

In Fig. 21 we have again used the Snowdon fields, this time with the correction sextupoles used to reduce the average chromaticity at small $\delta p/p$. With this correction, the region of essentially constant tune extends from $\delta p/p = -0.37\%$ to $\delta p/p = 0.35\%$.

Measurement results from the Magnet Test Facility on 16 dipoles[5] were used to estimate means and standard deviations of the distributions of values of the multipole components. Figure 22 shows the results when the average values are used for all multipole components, quadrupole through 30-pole, in every dipole. Correction quadrupoles and sextupoles have been used to reduce the tune splitting and chromaticity to reasonable levels. The required corrections are all less than 20% of the specified capabilities.[2]

For the last example, shown in Fig. 23, the multipole components for each of the 774 dipoles were selected randomly from the normal distributions defined by the means and standard deviations of the 16-magnet sample. The distributions were truncated at 3σ. There were no significant differences between this example and the previous one.

I want to thank Don Edwards, Mike Harrison, and Sho Ohnuma for many valuable discussions and suggestions concerning the work reported here.

References

[1]K. G. Steffan, **High Energy Beam Optics** (Interscience, New York, 1965).

[2]A Report on the Design of the Fermi National Accelerator Laboratory Superconducting Accelerator, May 1979.

[3]E. D. Courant and H. S. Snyder, Ann. Phys. 3, 1 (1958)

[4]M. Harrison, private communication.

[5]The data on the 16 dipoles were made available to me by Sho Ohnuma in mid-December 1979.

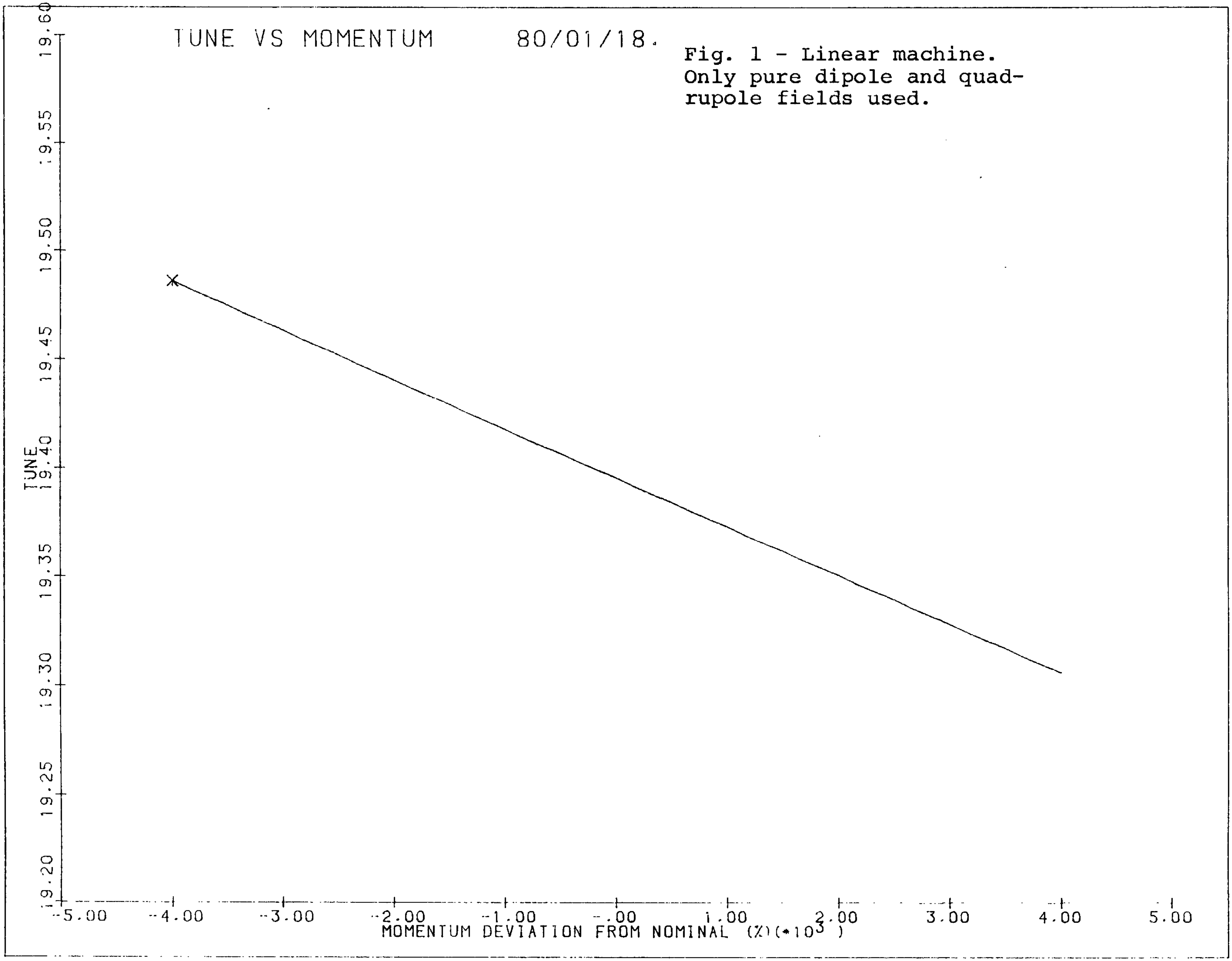


Fig. 1 - Linear machine.
Only pure dipole and quad-
rupole fields used.

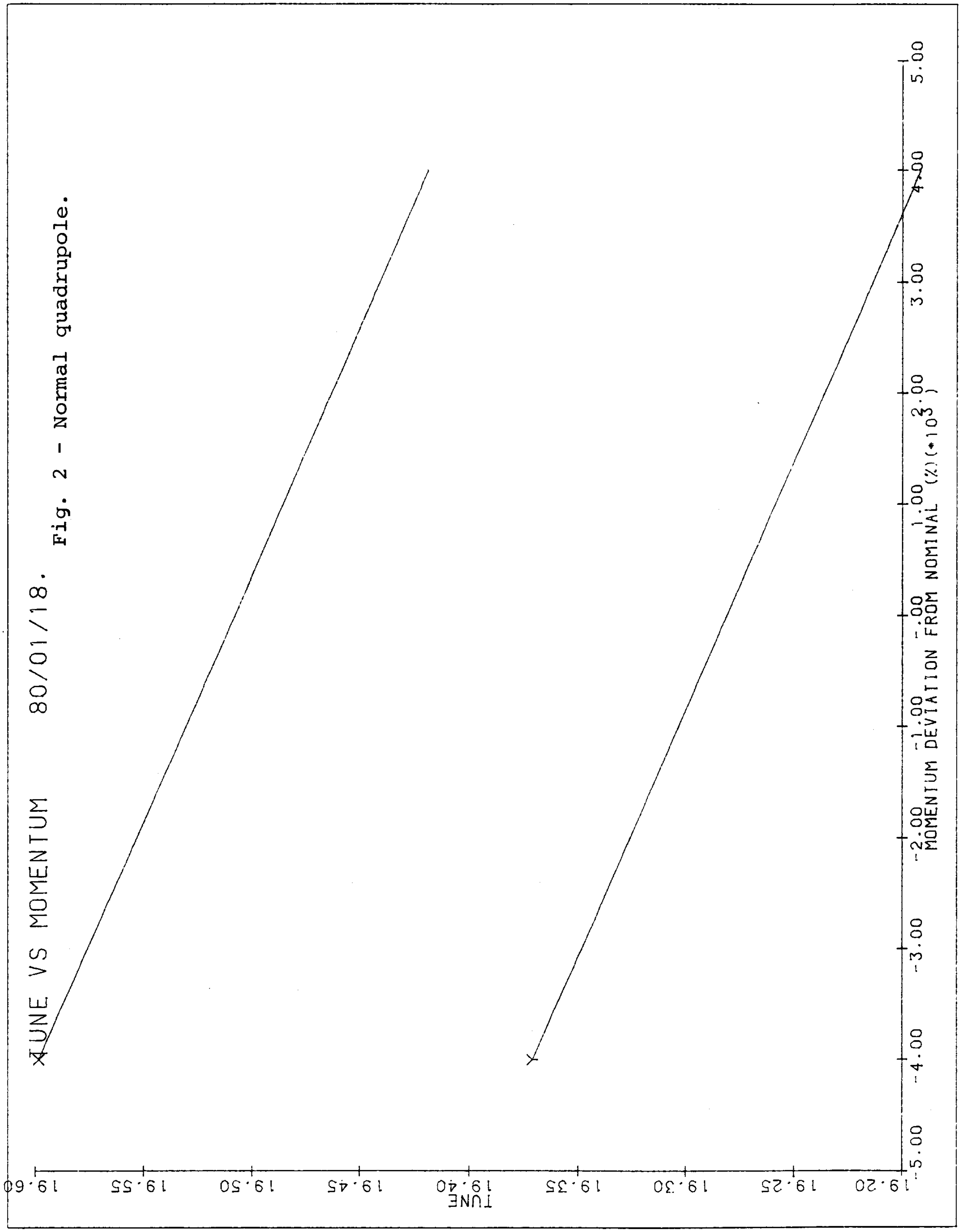


Fig. 2 - Normal quadrupole.

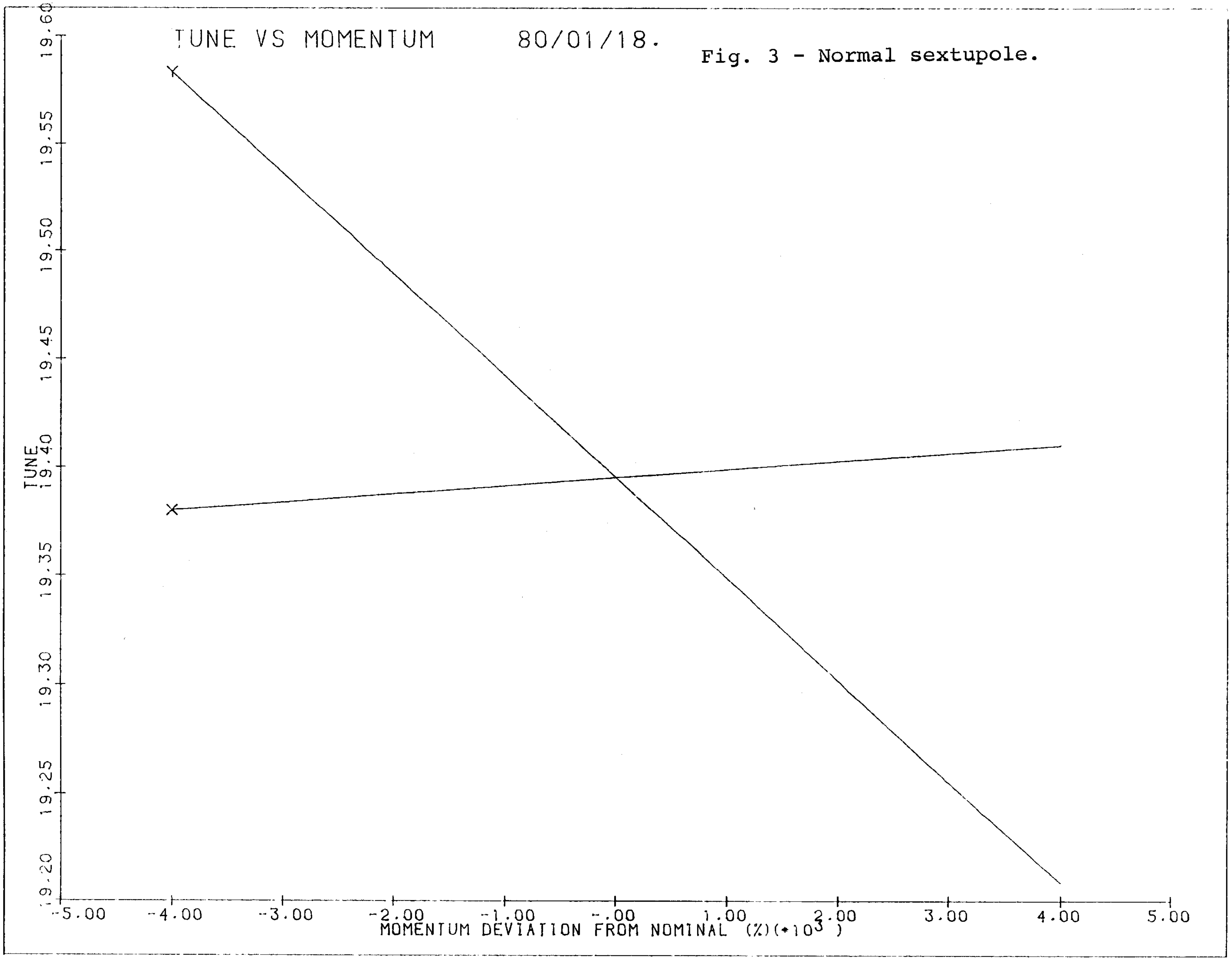


Fig. 3 - Normal sextupole.

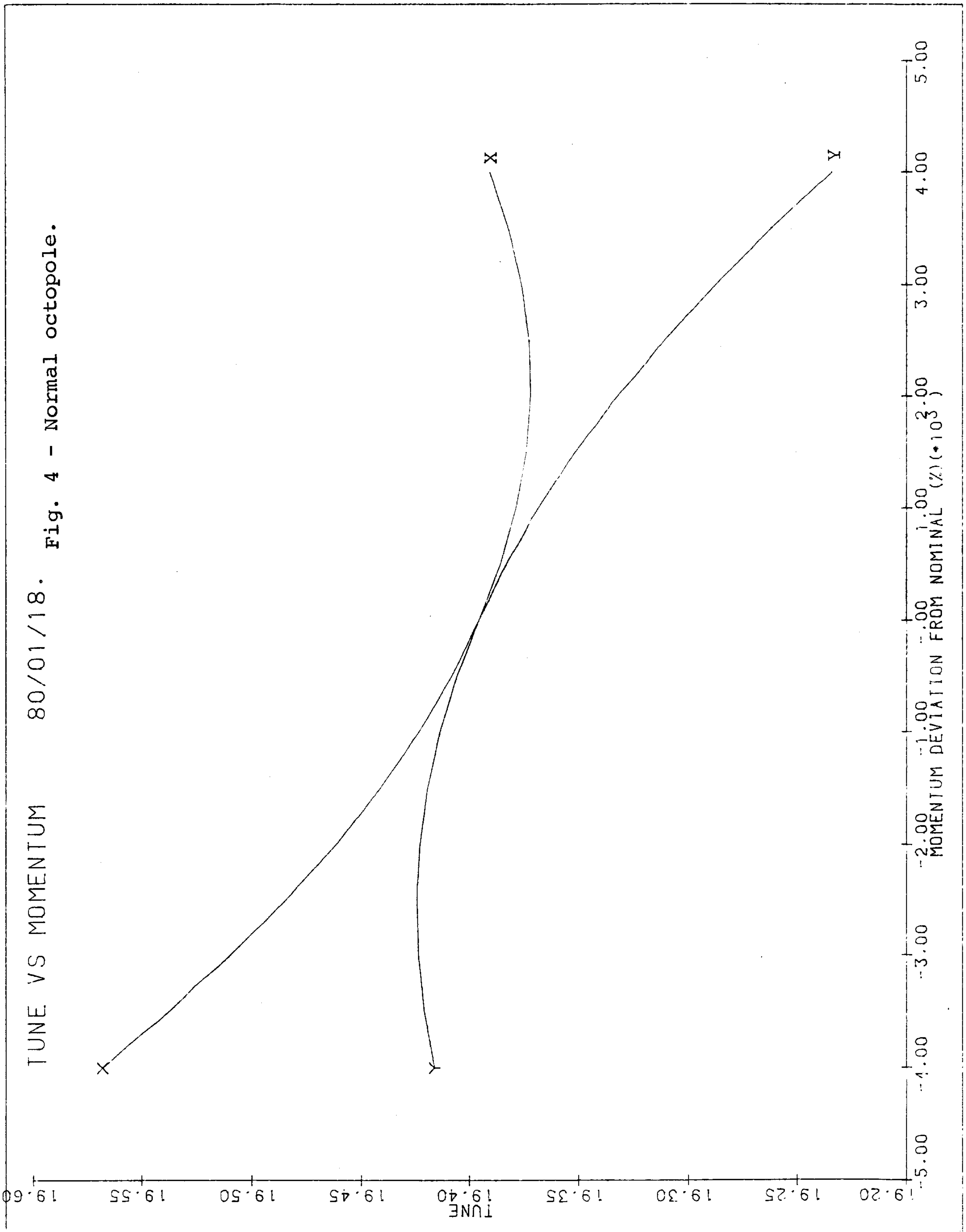


Fig. 4 - Normal octopole.

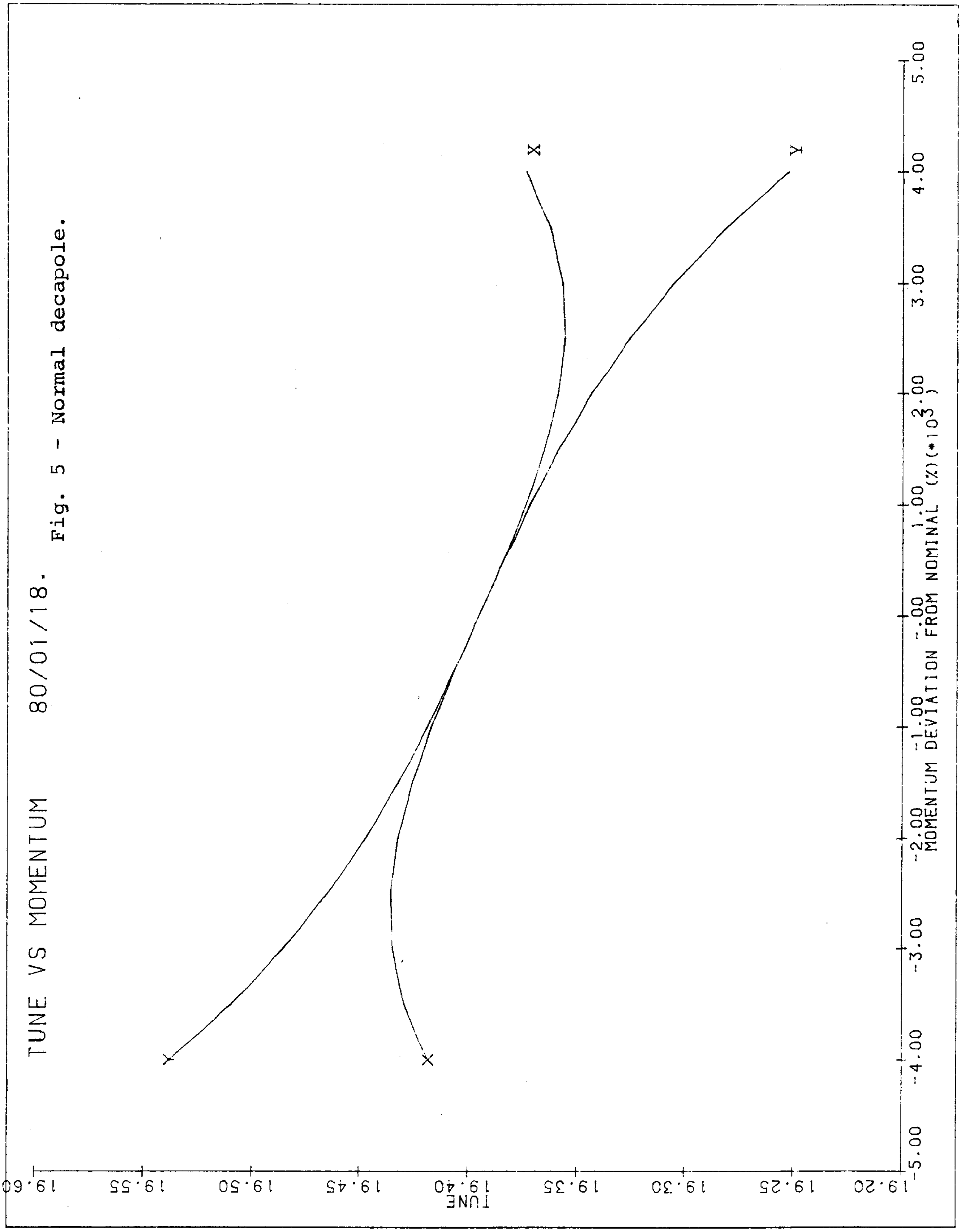


Fig. 5 - Normal decapole.

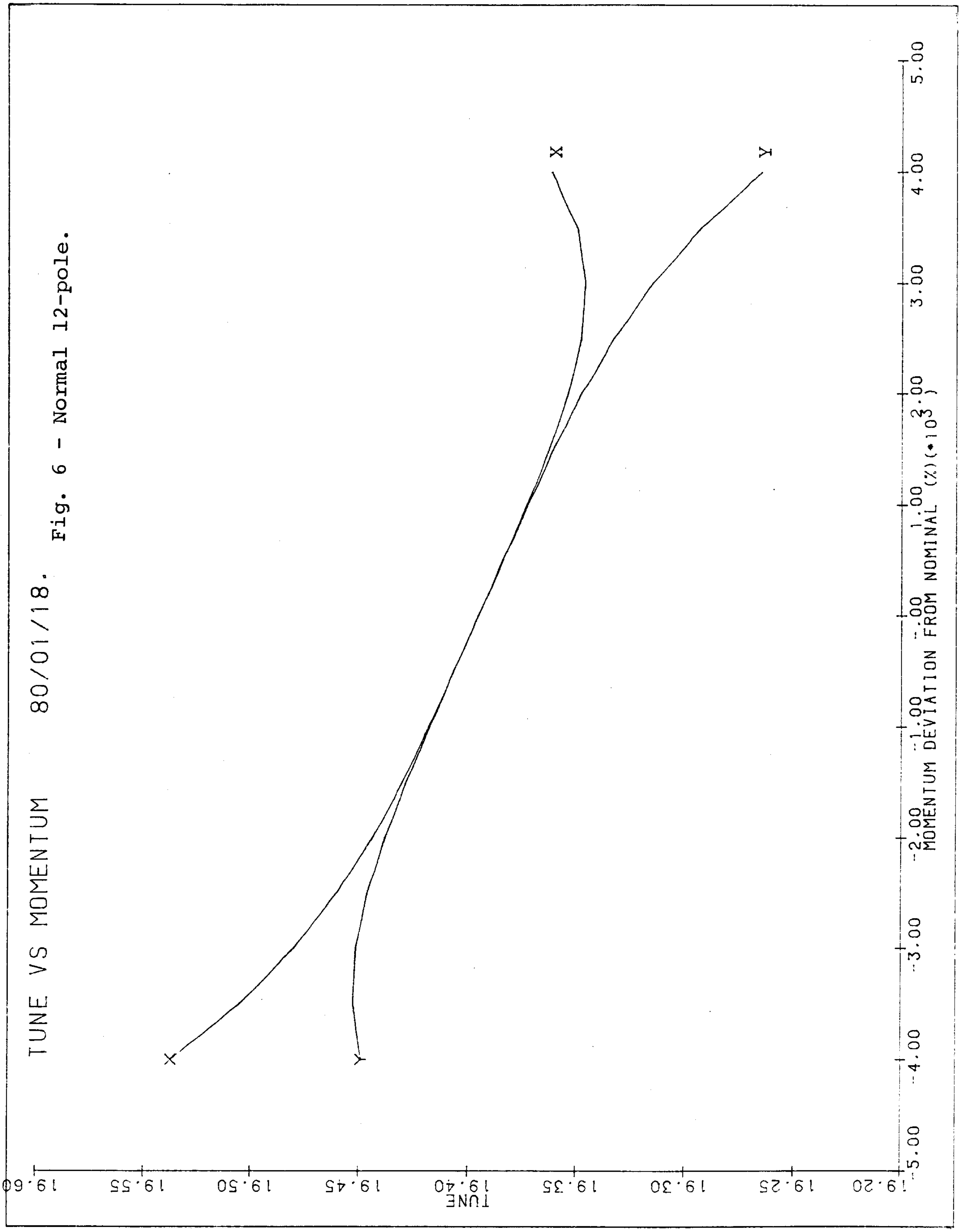


Fig. 6 - Normal 12-pole.

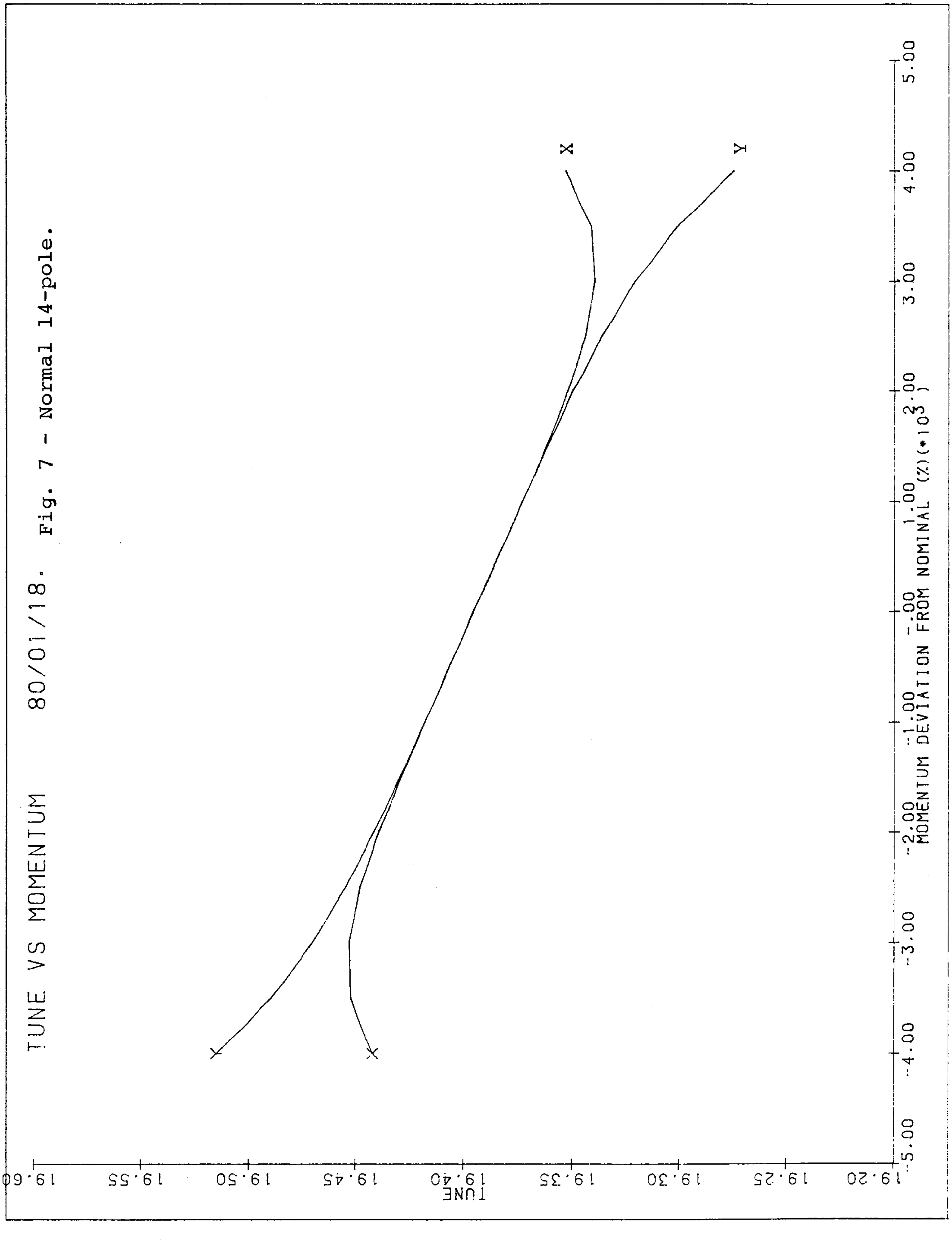


Fig. 7 - Normal 14-pole.

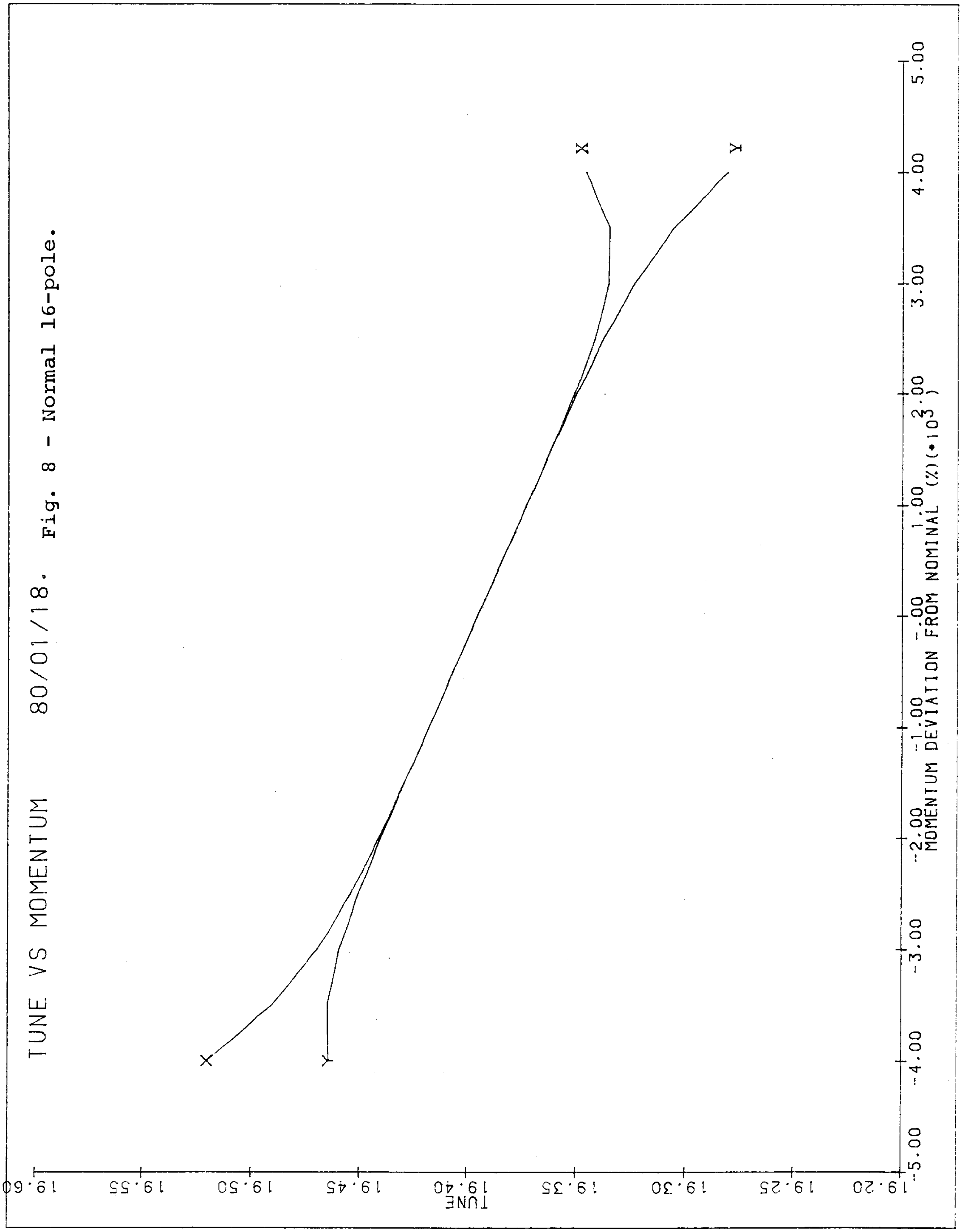


Fig. 8 - Normal 16-pole.

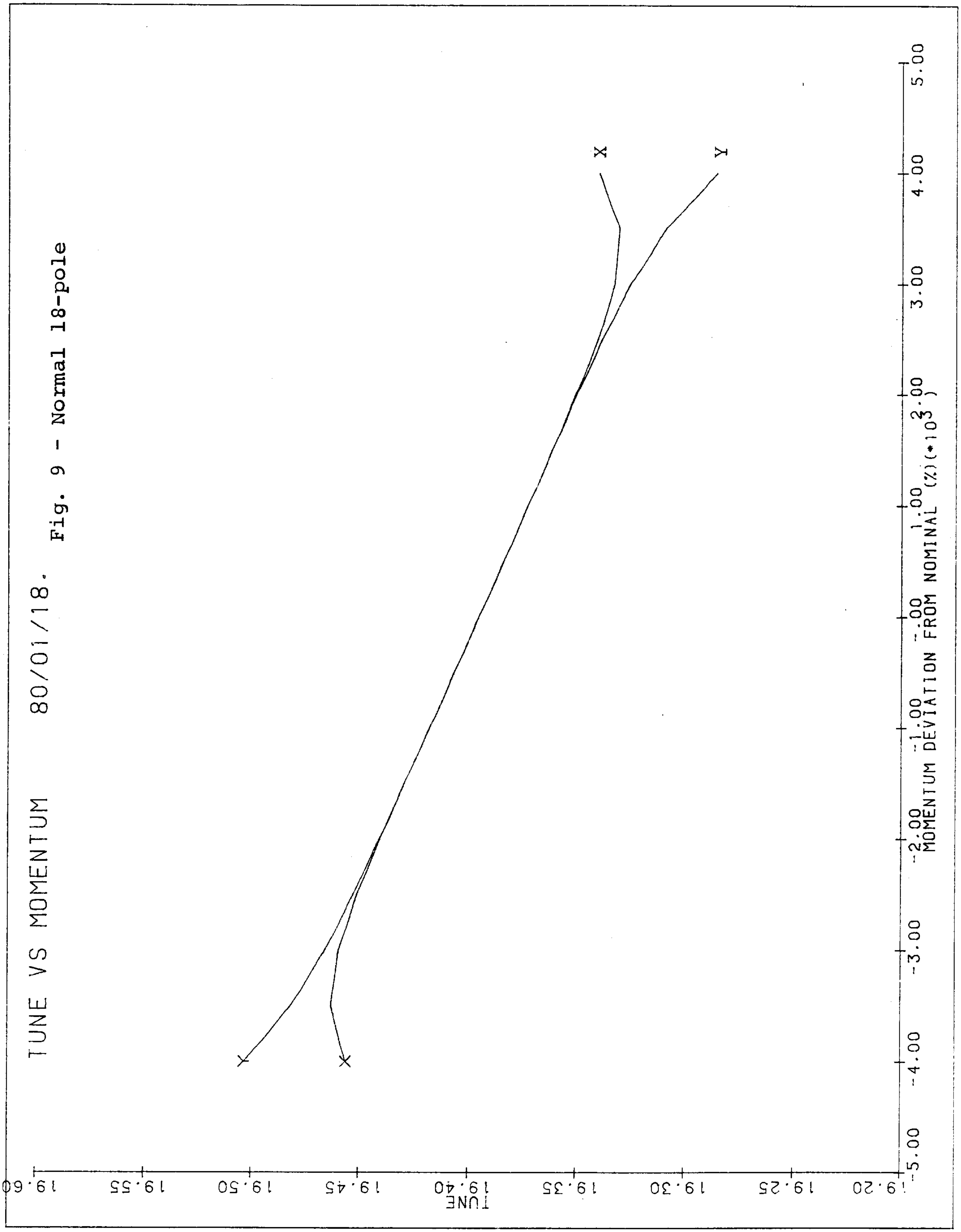


Fig. 9 - Normal 18-pole

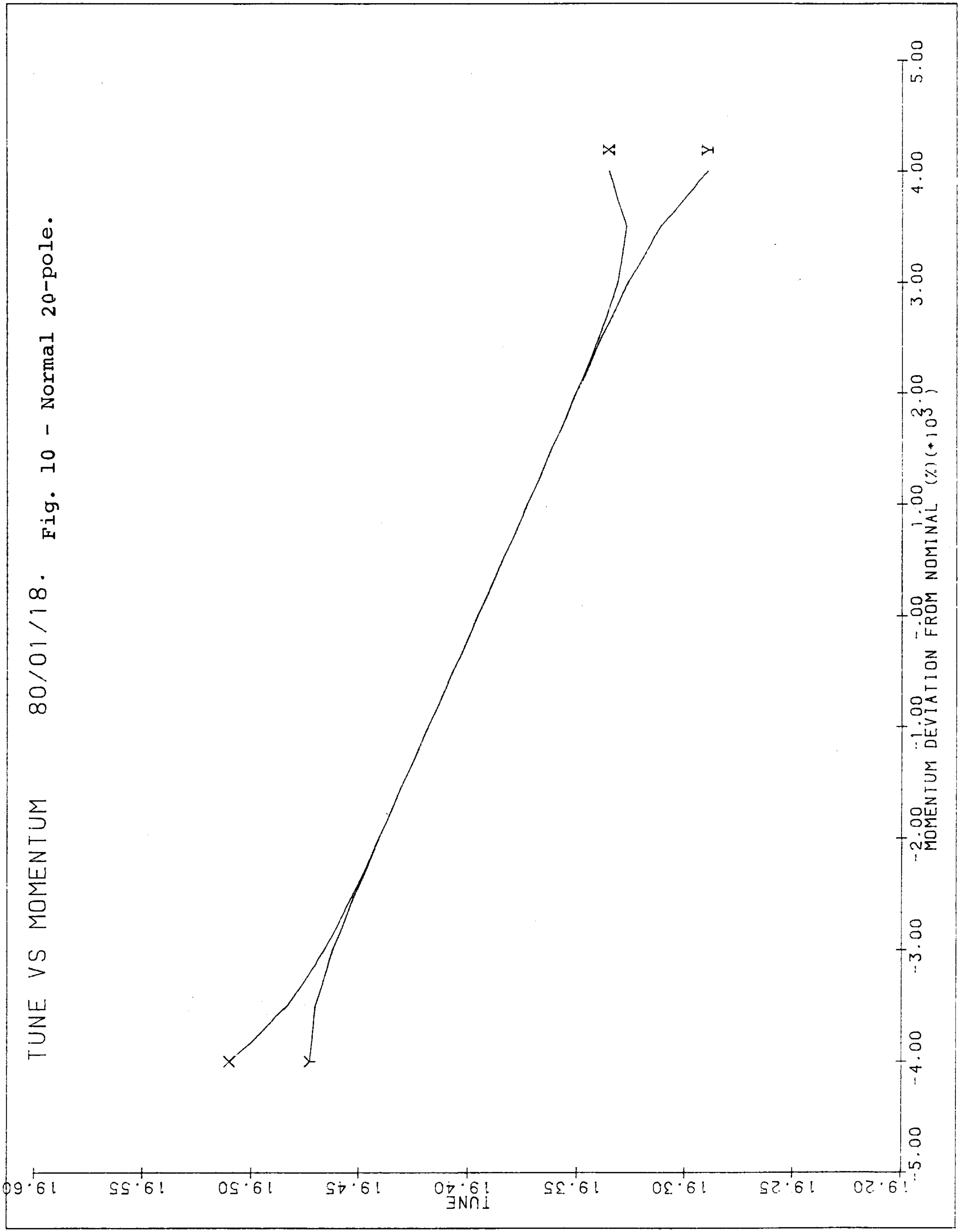
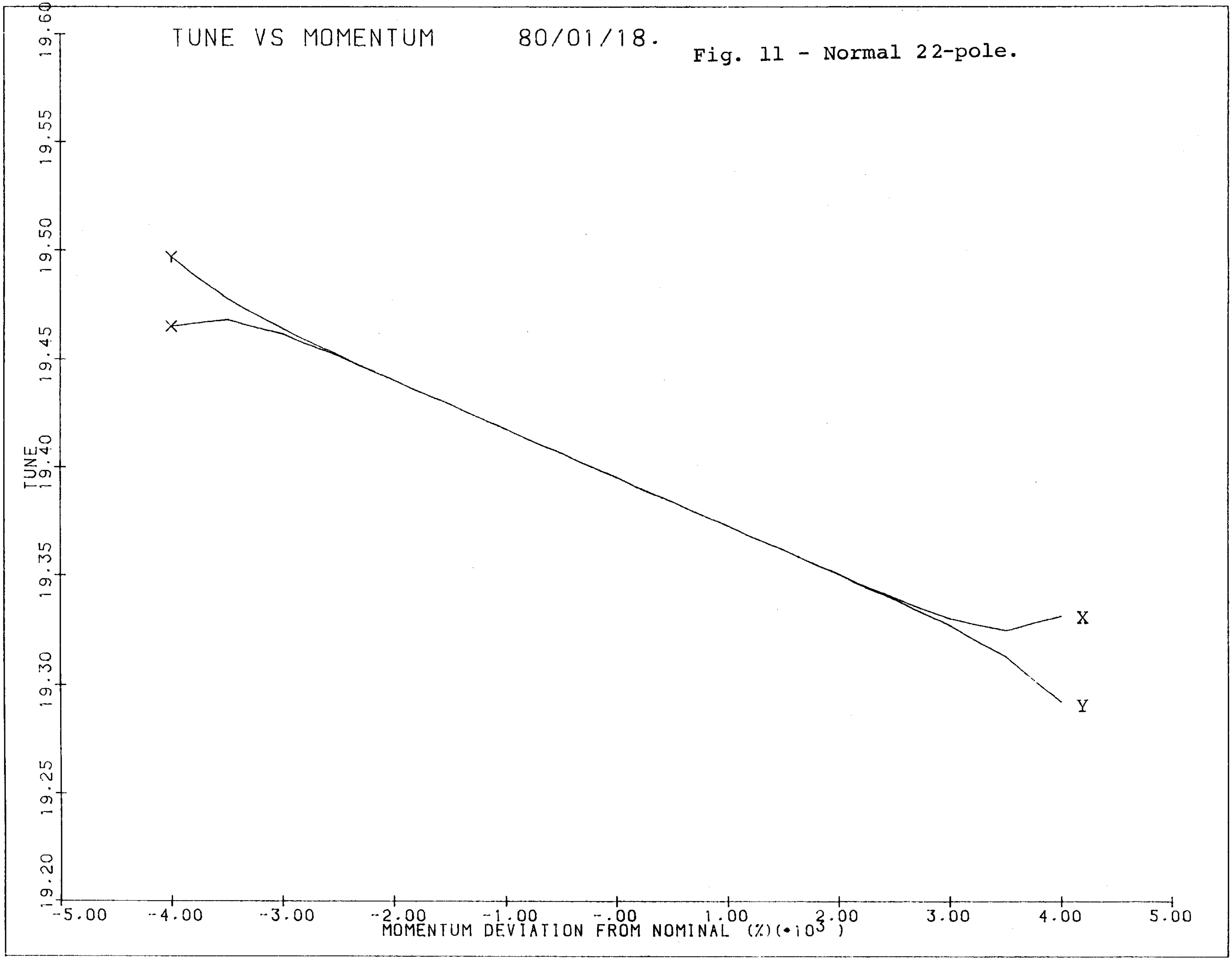


Fig. 10 - Normal 20-pole.

Fig. 11 - Normal 22-pole.



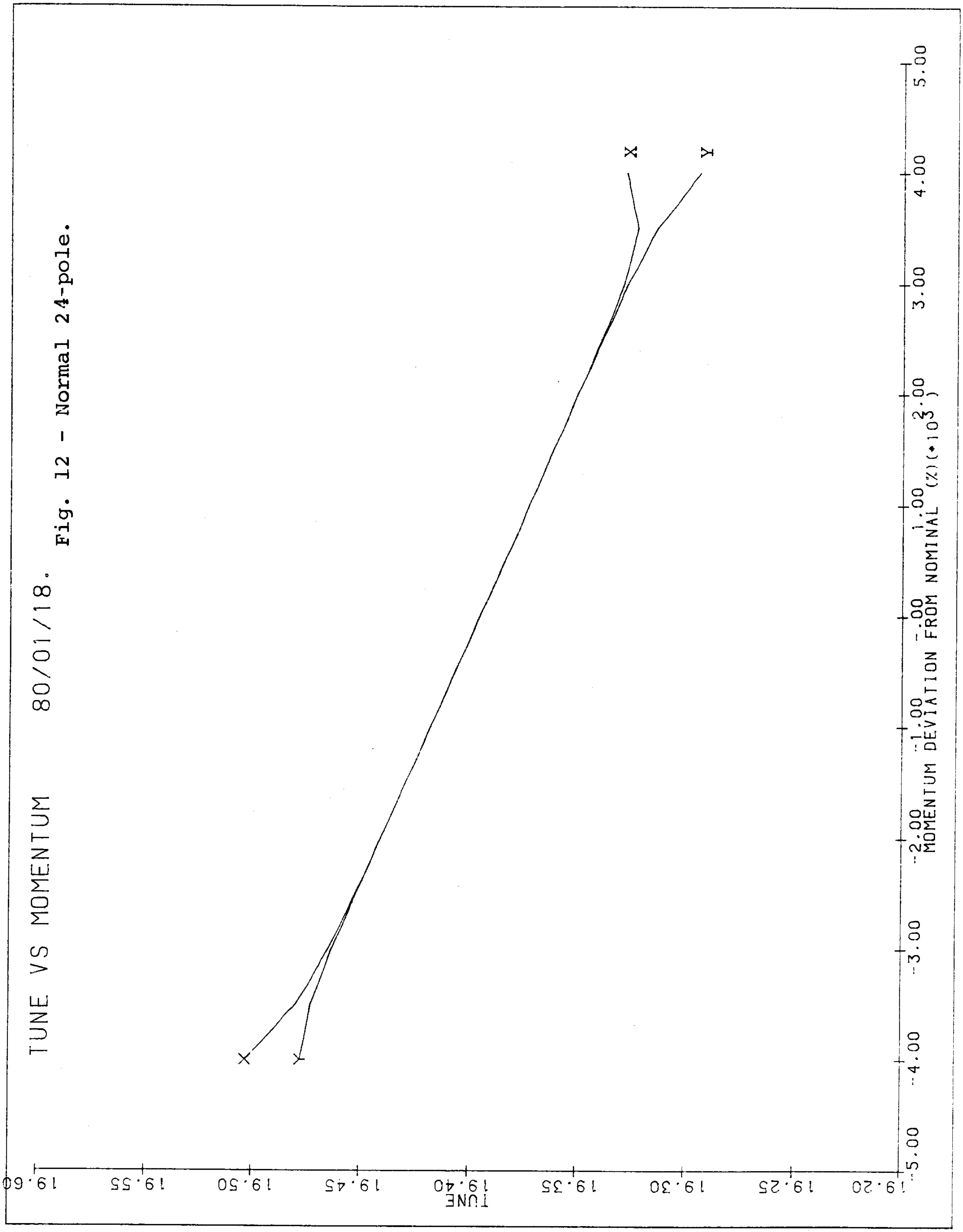


Fig. 12 - Normal 24-pole.

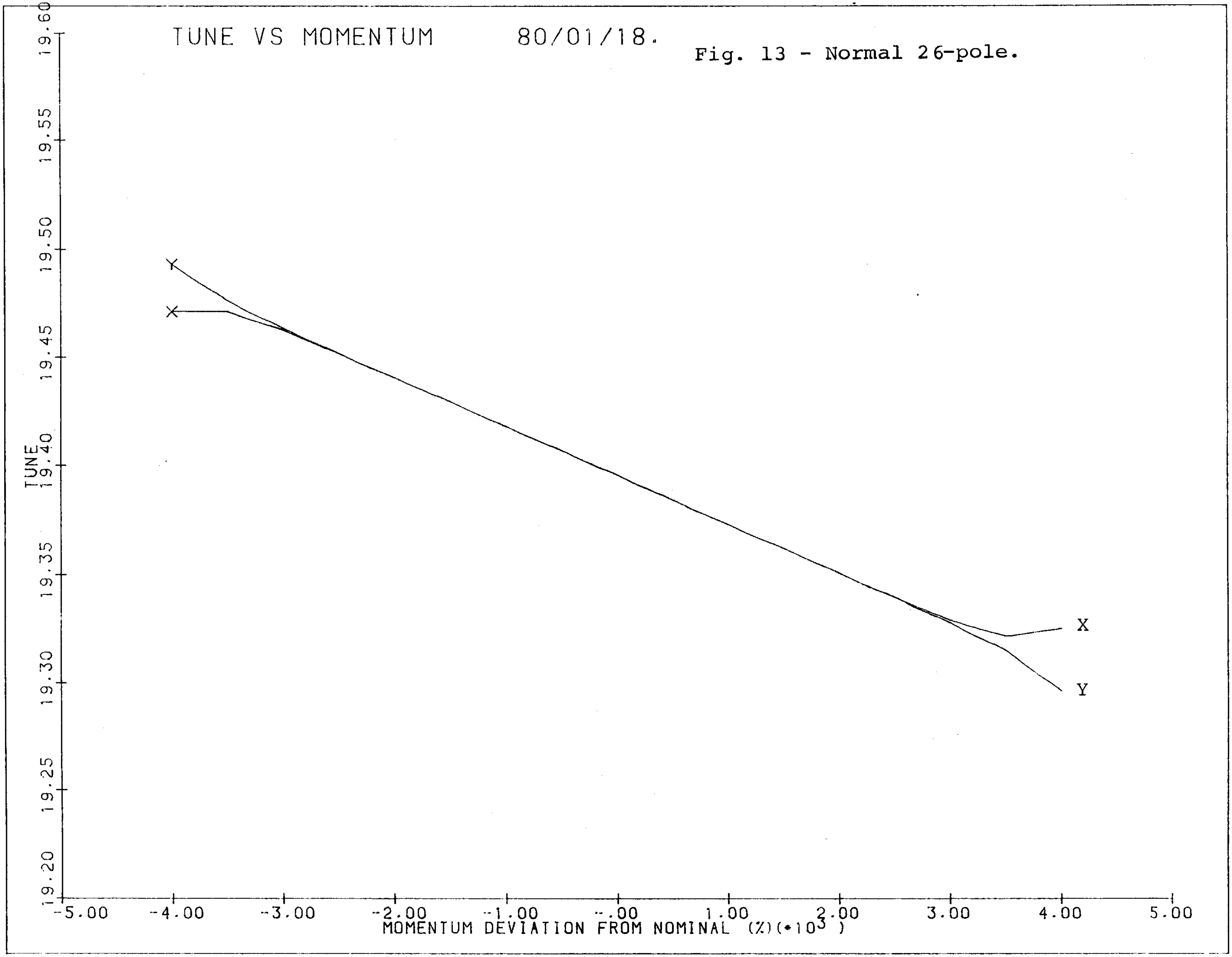


Fig. 13 - Normal 26-pole.

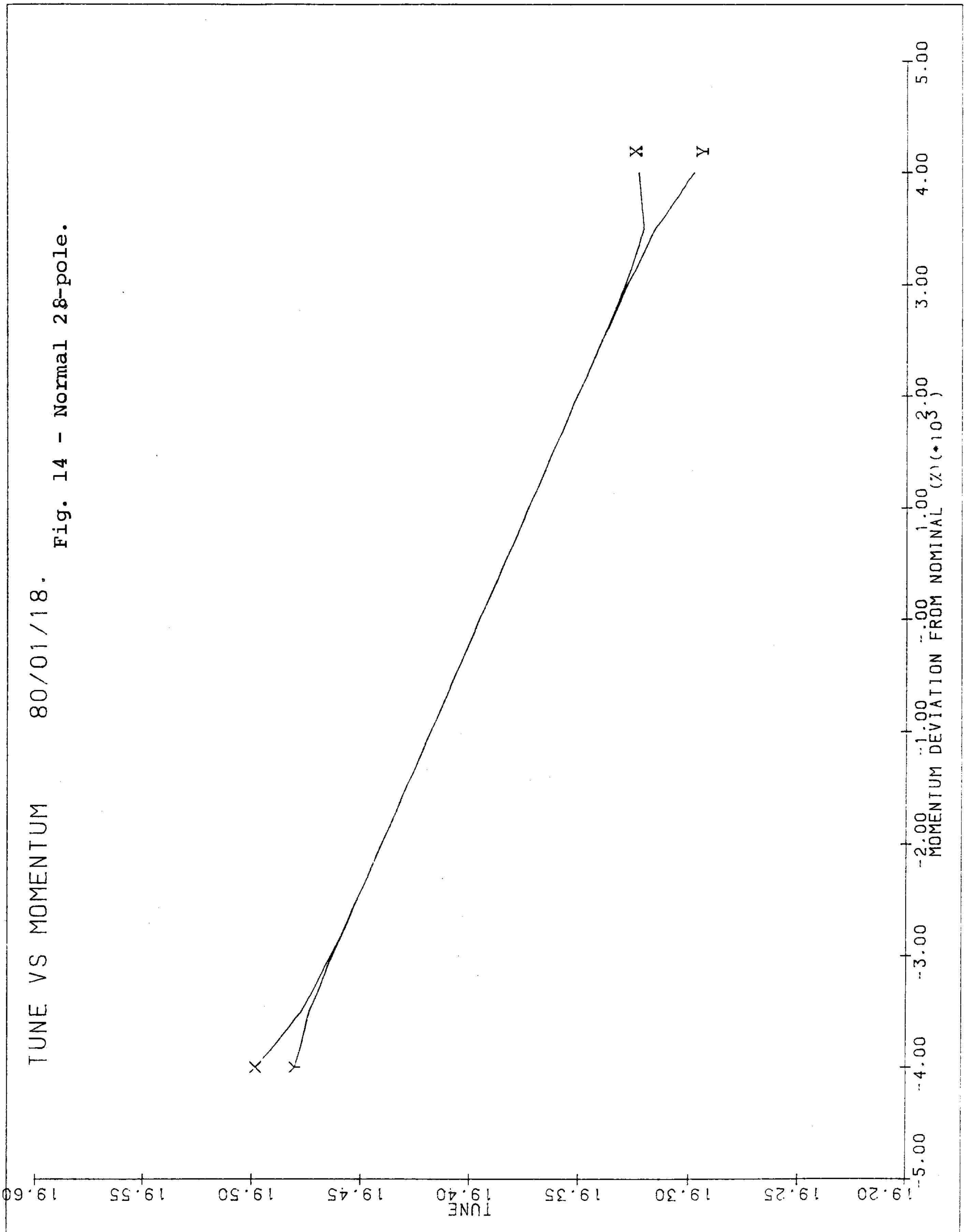


Fig. 14 - Normal 28-pole.

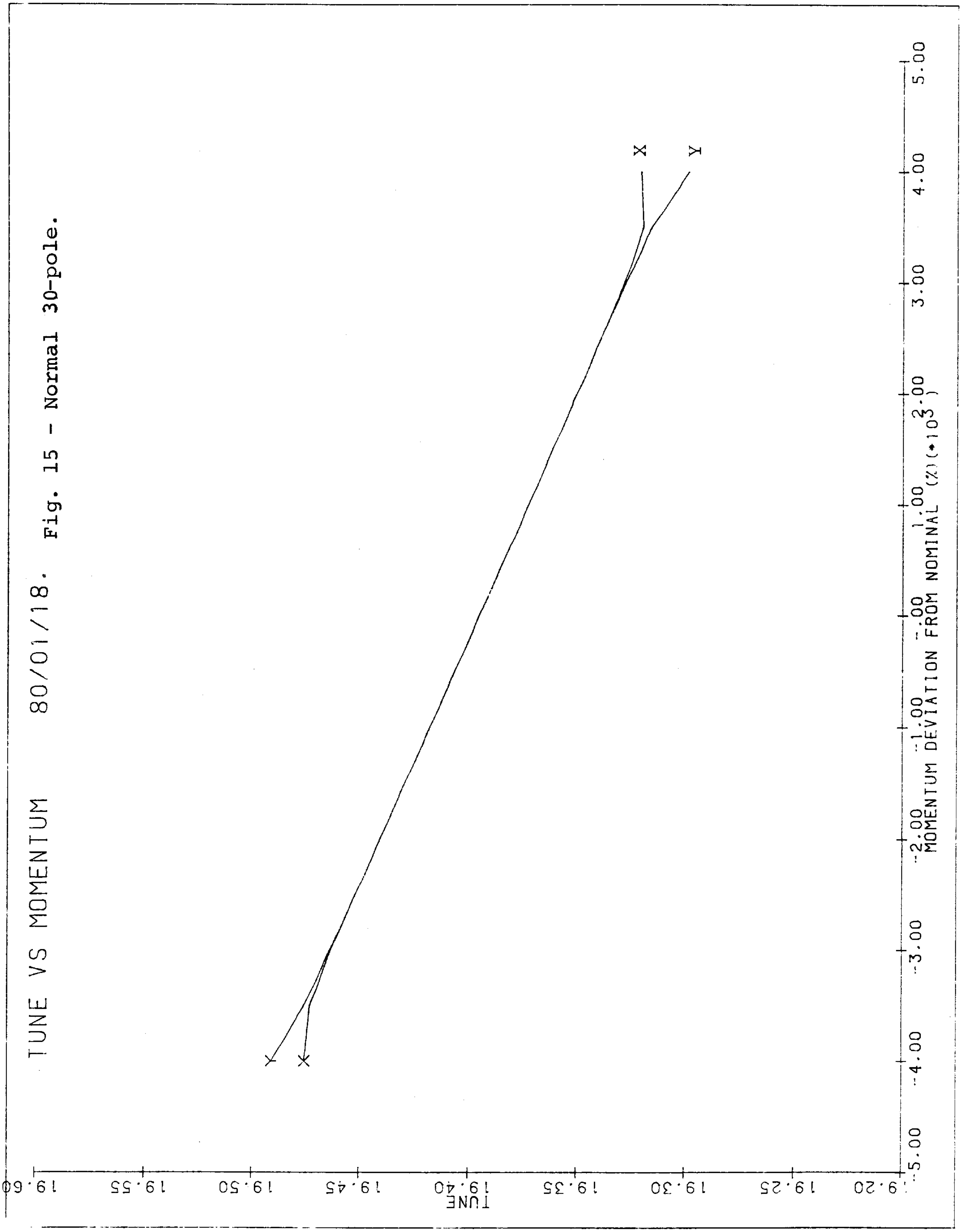


Fig. 15 - Normal 30-pole.

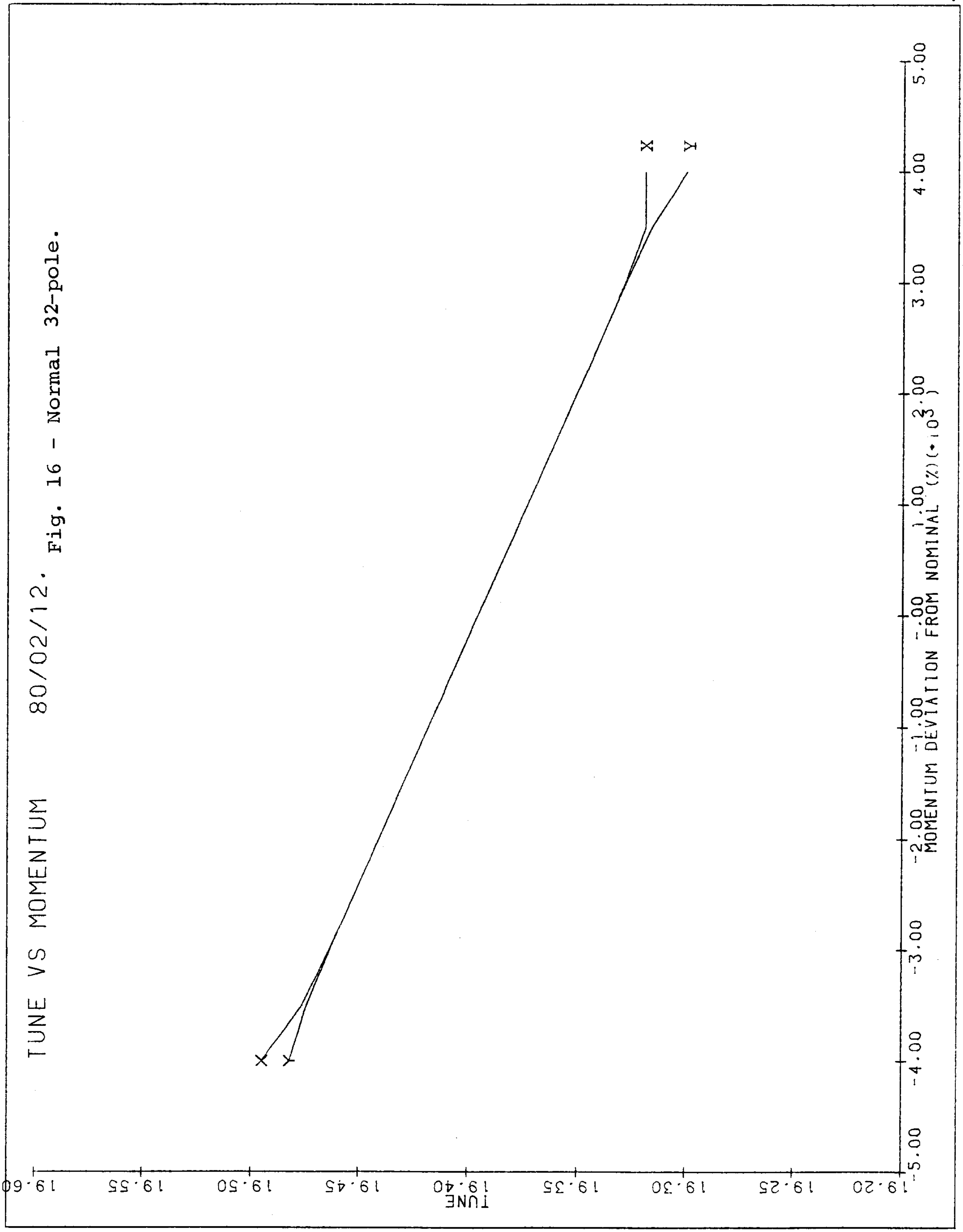


Fig. 16 - Normal 32-pole.

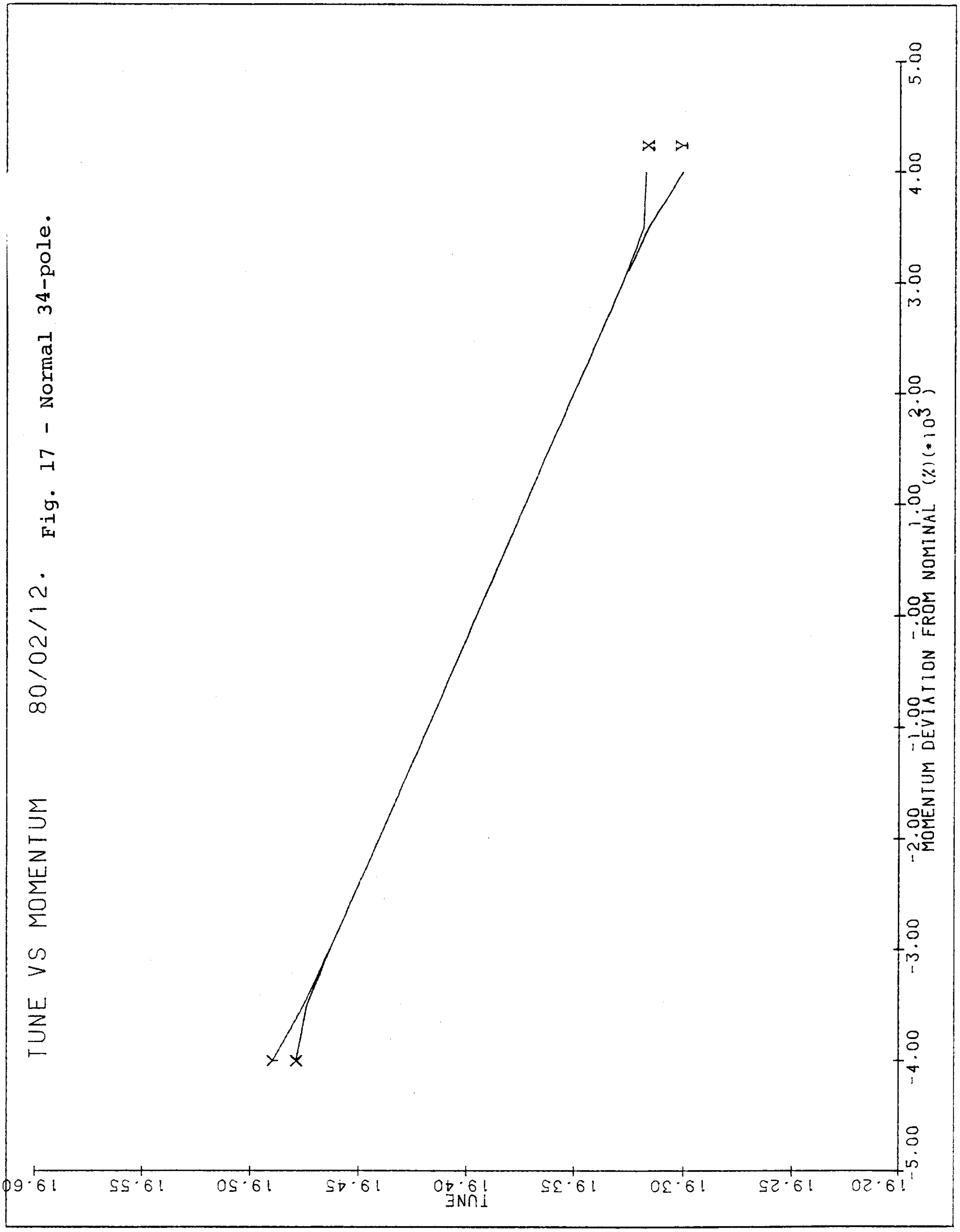


Fig. 17 - Normal 34-pole.

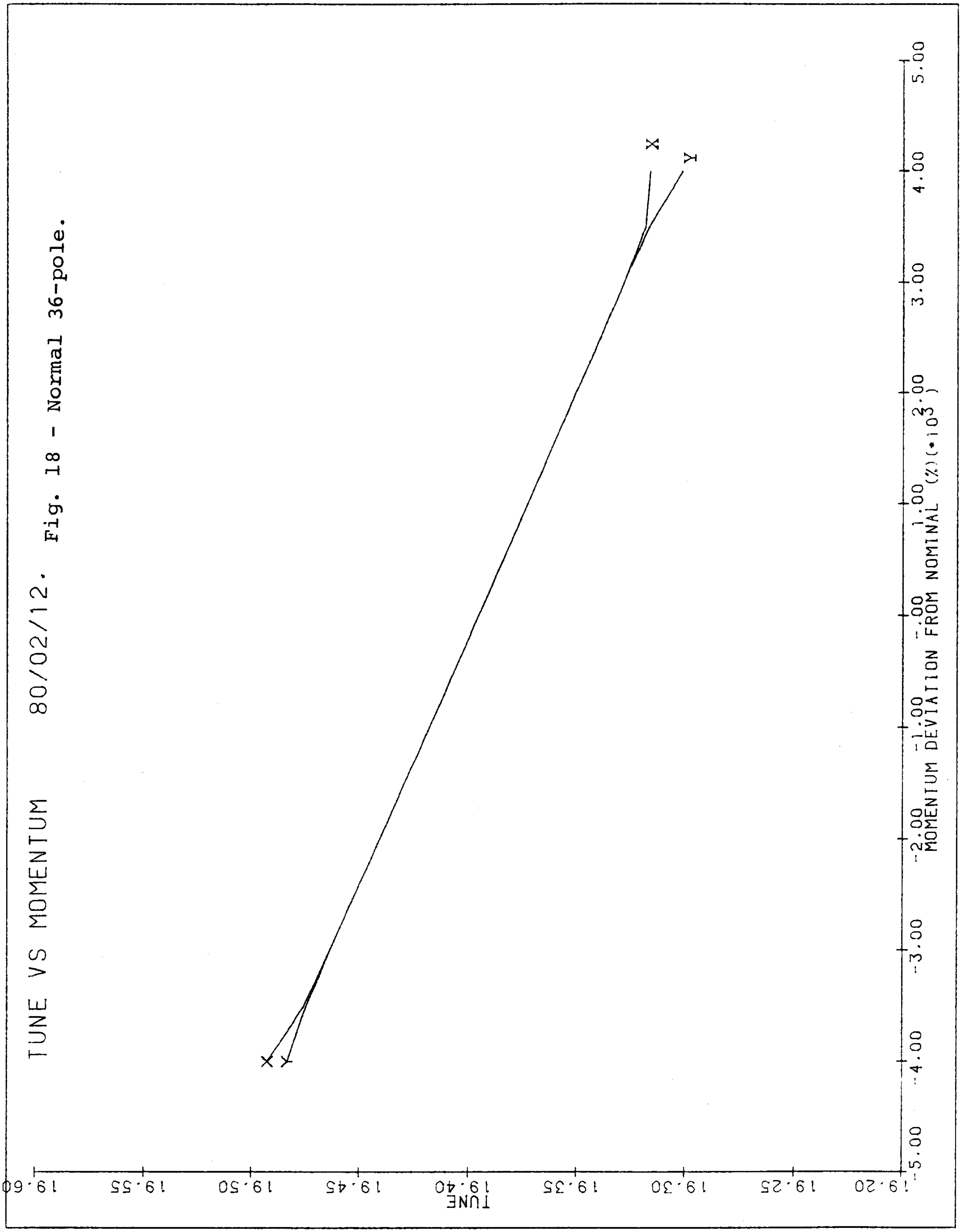


Fig. 18 - Normal 36-pole.

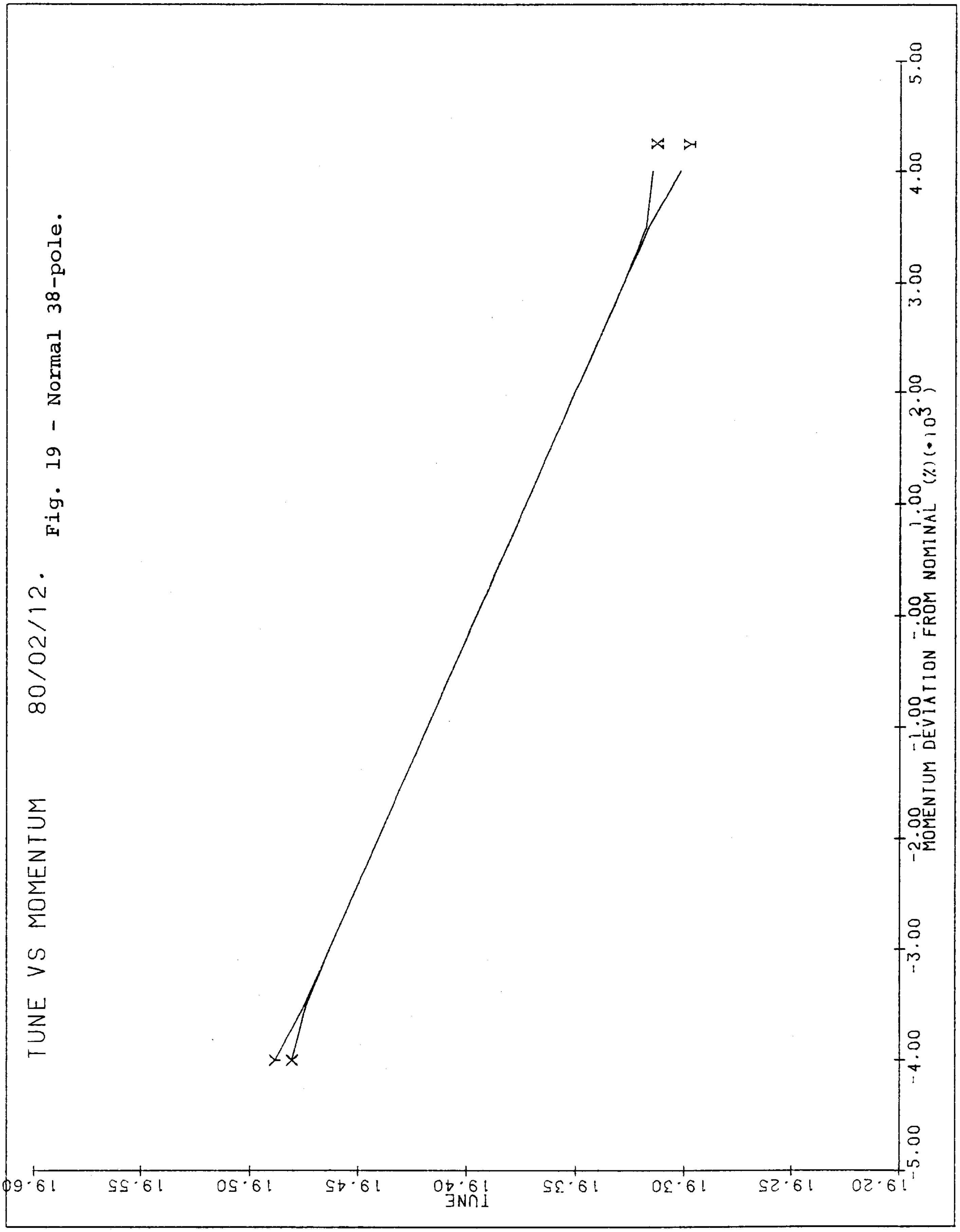


Fig. 19 - Normal 38-pole.

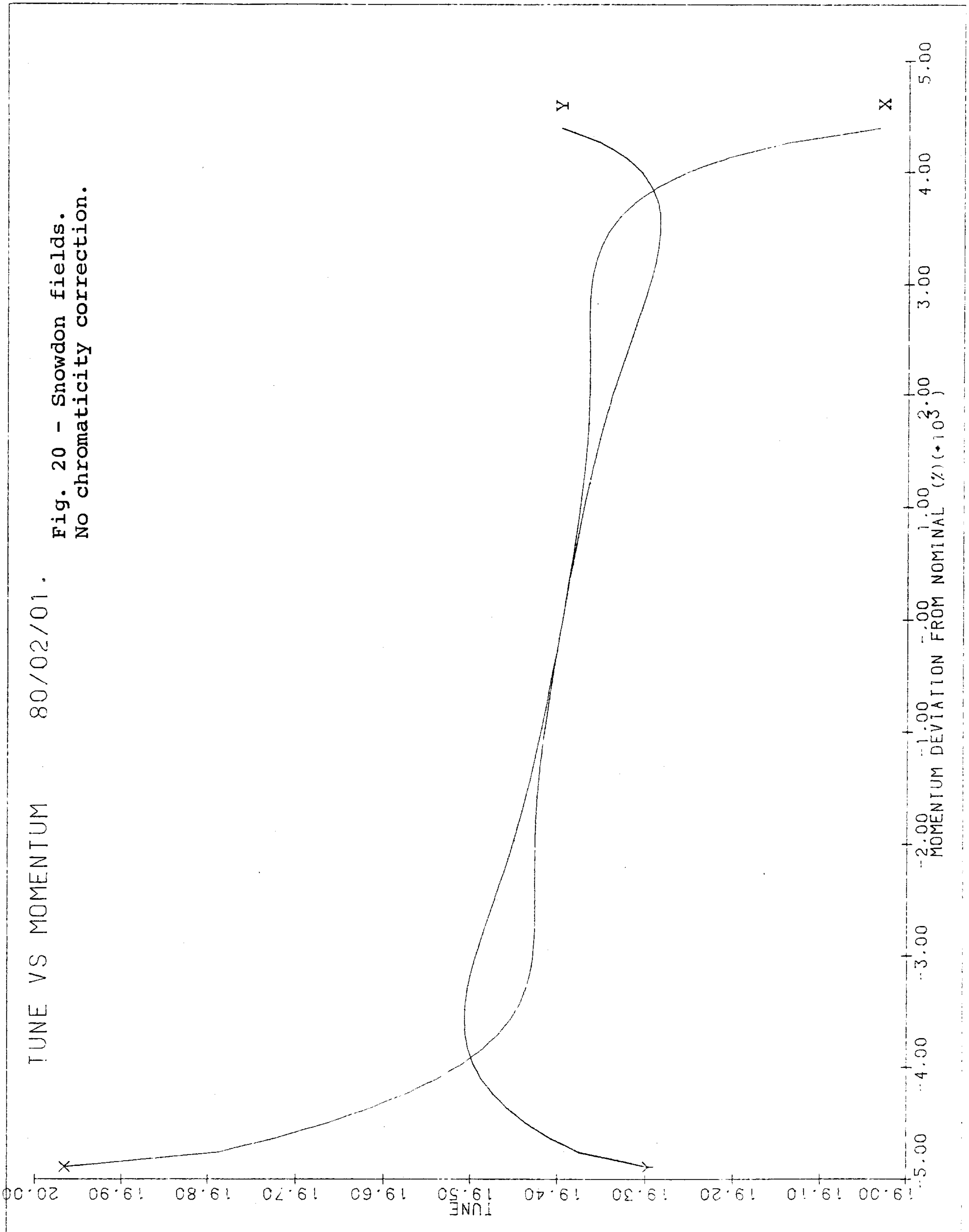


Fig. 20 - Snowdon fields.
No chromaticity correction.

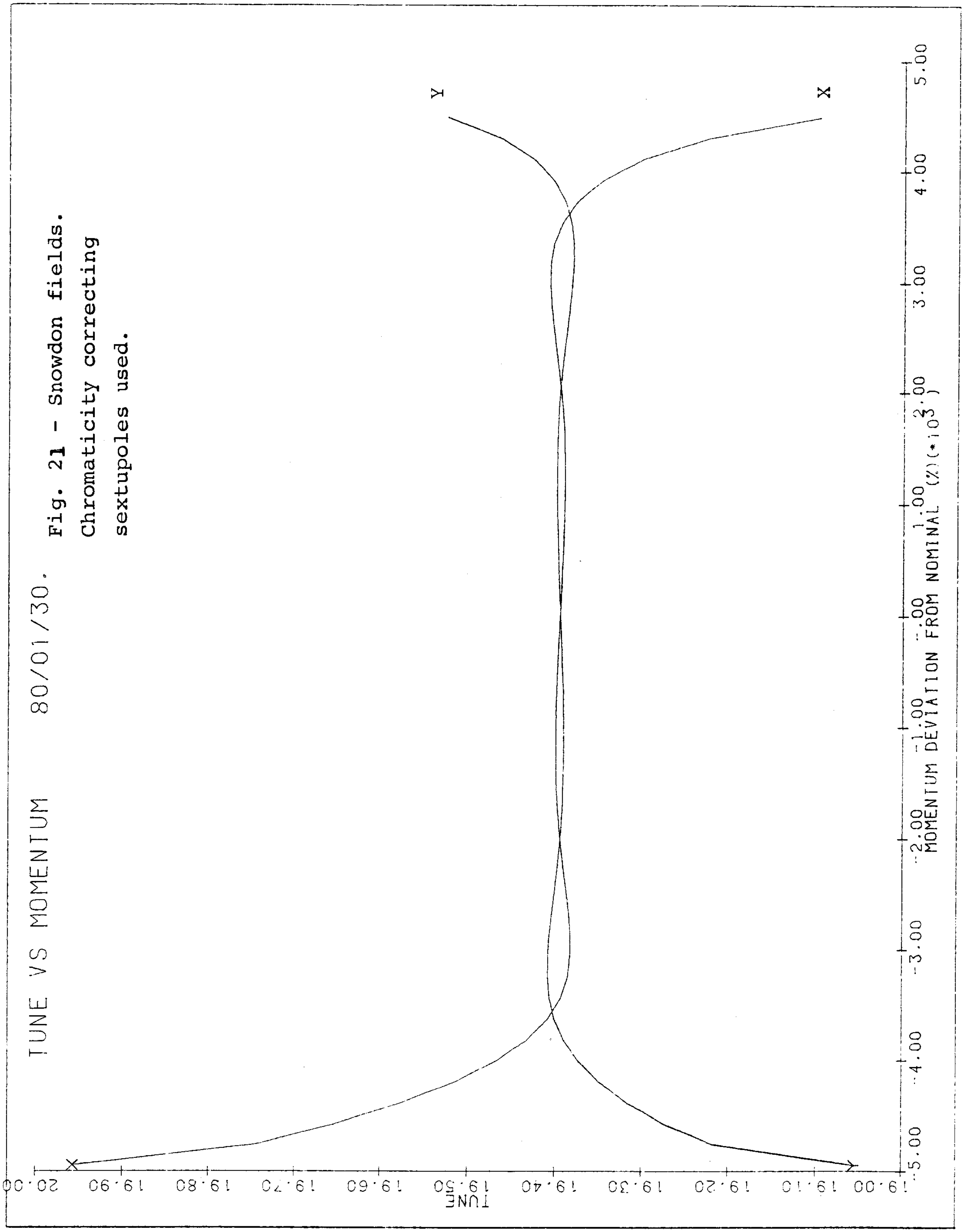


Fig. 21 - Snowdon fields.
Chromaticity correcting
sextupoles used.

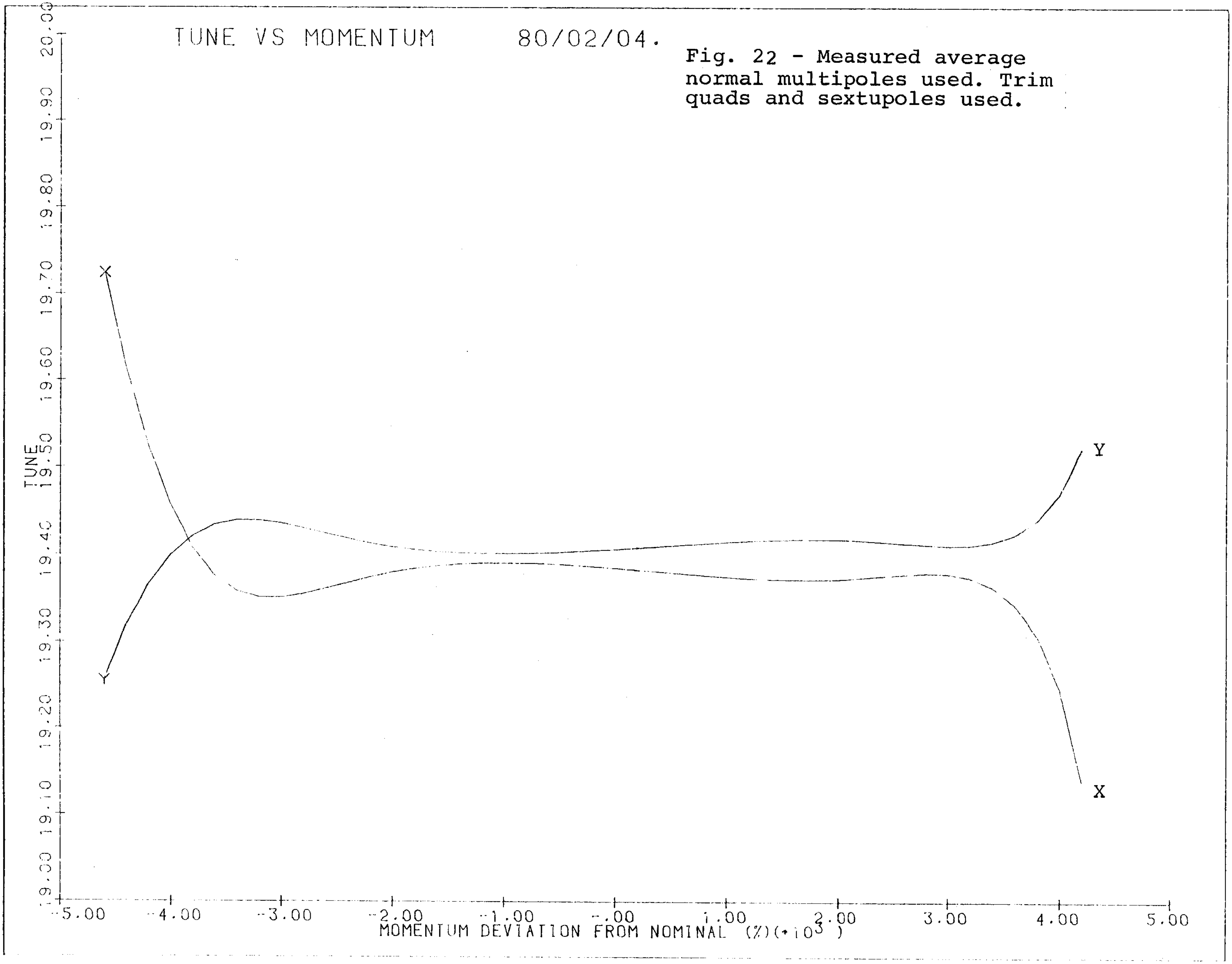


Fig. 22 - Measured average normal multipoles used. Trim quads and sextupoles used.

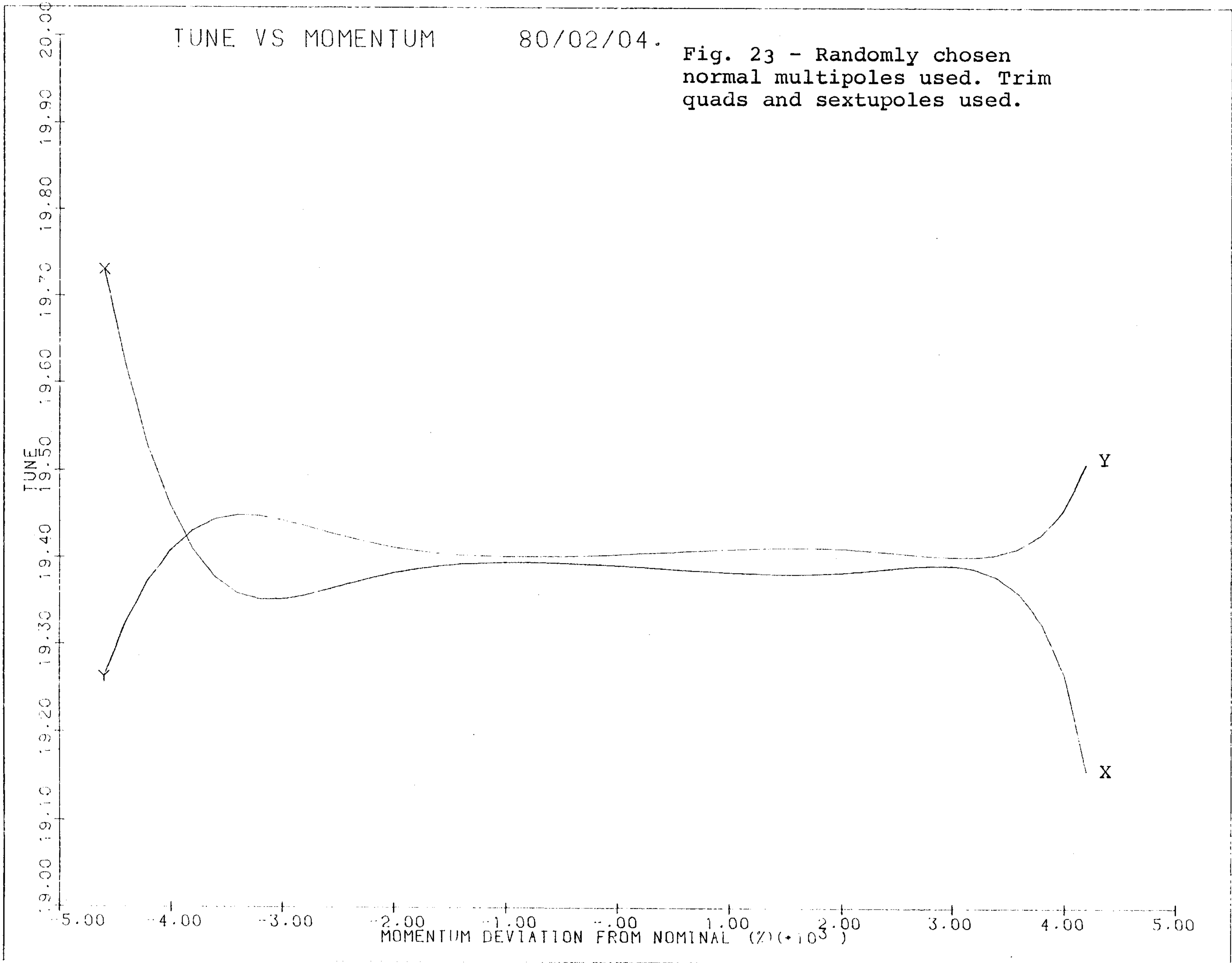


Fig. 23 - Randomly chosen normal multipoles used. Trim quads and sextupoles used.